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Why the Frequency Spectrum is meaningful, talk about some Engineering Applications of Signal Spectrum.

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Any analog signal is a mixture of number of sinusoidal signals of various frequencies. The various sinusoidal component frequencies of a signal are called spectrum. A sinusoidal signal is a complex quantity. So it has magnitude and phase. Hence frequency spectrum consists of magnitude plot and phase plot. The **frequency spectrum** of a time-domain [signal](http://en.wikipedia.org/wiki/Signal_(electronics)) is a representation of that signal in the [frequency domain](http://en.wikipedia.org/wiki/Frequency_domain). The frequency spectrum can be generated via a [Fourier transform](http://en.wikipedia.org/wiki/Fourier_transform) of the signal, and the resulting values are usually presented as [amplitude](http://en.wikipedia.org/wiki/Amplitude) and [phase](http://en.wikipedia.org/wiki/Phase_(waves)), both plotted versus [frequency](http://en.wikipedia.org/wiki/Frequency). Any signal that can be represented as amplitude that varies with time has a corresponding frequency spectrum. When the physical phenomena are represented in the form of a frequency spectrum, certain physical descriptions of their internal processes become much simpler. Often, the frequency spectrum clearly shows harmonics, visible as distinct spikes or lines that provide insight into the mechanisms that generate the entire signal. In engineering, the **frequency domain** is the domain for analysis of [mathematical functions](http://en.wikipedia.org/wiki/Mathematical_function) or [signals](http://en.wikipedia.org/wiki/Signal_(information_theory)) with respect to [frequency](http://en.wikipedia.org/wiki/Frequency), rather than time. Put simply, a [time-domain](http://en.wikipedia.org/wiki/Time-domain) graph shows how a signal changes over time, whereas a frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies. A frequency-domain representation can also include information on the phase shift that must be applied to each [sinusoid](http://en.wikipedia.org/wiki/Sinusoid) in order to be able to recombine the frequency components to recover the original time signal. A given function or signal can be converted between the time and frequency domains with a pair of mathematical [operators](http://en.wikipedia.org/wiki/Operator_(mathematics)) called a [transform](http://en.wikipedia.org/wiki/Transform_(mathematics)). An example is the [Fourier transform](http://en.wikipedia.org/wiki/Fourier_transform), which decomposes a function into the sum of a (potentially infinite) number of [sine wave](http://en.wikipedia.org/wiki/Sine_wave) frequency components. The 'spectrum' of frequency components is the frequency domain representation of the signal. The [inverse Fourier transform](http://en.wikipedia.org/wiki/Inverse_Fourier_transform) converts the frequency domain function back to a time function.

Over 80% of all DSP applications require some form of frequency domain processing and there are many techniques for performing this kind of operation. Frequency domain techniques can often provide information that is difficult or sometimes impossible to realize in the time domain. All signals have a frequency domain representation. Signals can be transformed between the time and the frequency domain through various transforms. The signals can be processed within these domains and each process in one domain has a corollary in the other, as shown:



The most important process translation between the time and frequency domain is that convolution in the time domain is the equivalent to multiplication in the frequency domain and vice versa. Using the [Fourier transform](http://www.bores.com/courses/intro/freq/3_ft.htm), any signal can be analyzed into its frequency components. Every signal has a frequency spectrum. The signal defines the spectrum and the spectrum defines the signal. We can move back and forth between the time domain and the frequency domain without losing information.

**Fourier Transform:**

The Fourier Transform (FT) is derived from the definition of the Fourier Series (FS). Consider, for example, the periodic complex signal *gTo*(*t*) with period*T*0 = 2π/*ω*0. The exponential FS of that signal allows the representation of *gTo*(*t*) as

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Where the complex coefficients *D*n  are evaluated as



for any period*T*0 . For convenience, we will use the period – *T*0 /2 ≤ t ≤ *T*0 /2,

which gives

,

Note that the magnitudes and phases of the coefficients *D*n  are, respectively, known as the amplitude and phase spectrums of *gTo*(*t*), or

| *D*n  | is amplitude spectrum of *gTo*(*t*) , and

∠ *D*n is phase spectrum of *gTo*(*t*).

Now, let us separate the different periods of *gTo*(*t*) with zeros such that the different periods of *gTo*(*t*) move away from each other (by holding one period of *gTo*(*t*) and increasing the period duration*T*0  until it becomes infinite). When *T*0 → ∞, 🡪 the signal *gTo*(*t*) can be renamed *g*(*t*) since it is no longer periodic but only contains one period of *gTo*(*t*). In this case, *ω*0 = 2π/*T*0 → 0, and the discrete-time signal represented by the coefficients *D*n vs. *n* becomes (in the limit) a continuous-time signal in terms of a new variable *ω* = *nω*0.Therefore,



can be written as

,

where *ω = nω*0, and *D*n = *G*(*nω*0)/*T*0 .

**Fourier Transform (FT) and Inverse Fourier Transform (IFT):**

The FT of signal *g*(*t*) is denoted F [*g*(*t*)] and is defined as , ............eq. A.

and the IFT of *G*(*ω*) is denoted F –1 [*G*(*ω*)] is defined as  ..............eq.B.

We say that *g*(*t*) and *G*(*ω*) for a FT pair, or *g*(*t*) ⇔ *G*(*ω*).

Notice that the exponent term in eq.A has a negative sign but no negative sign exists in the exponent in eq.B. Also, notice that the integration in eq.A is in terms of *t* and it is in terms of *ω* in eq.B.

Also notice that *G*(*ω*) in general is a complex signal that has both a magnitude |*G*(*ω*)| and a phase *θG*(*ω*) , or

.

**General Properties of the FT:**

1) Symmetry:

For **REAL** *g*(*t*) 🡺 *G*(–*ω*) = *G\**(*ω*)

|*G*(–*ω*)| = |*G\**(*ω*)| = |*G*(*ω*)|

*θG*(–*ω*) = – *θG*(*ω*)

For REAL *g*(*t*) with 🡺 *G*(*ω*) is PURLEY REAL

*g*(–*t*) = *g*(*t*) (even functions)

For REAL *g*(*t*) with 🡺 *G*(*ω*) is PURELY IMAGINARY

*g*(–*t*) = – *g*(*t*) (odd functions)

2) Existence of the FT:

For a signal *g*(*t*), if , 🡺 FT F [*g*(*t*)] exists.

The opposite is not necessarily true. This above existence condition comes from the fact that



Therefore, if the integration of the magnitude of a function is finite, then the FT integral is also finite and therefore, the FT exists.

3) Linearity:

If *g*(*t*) ⇔ *G*(*ω*) and *f*(*t*) ⇔ *F*(*ω*),then

*a⋅ g*(*t*) + *b⋅* *f*(*t*) ⇔ *a⋅ G*(*ω*) + *b⋅* *F*(*ω*).

**FT of Important Functions:**

1) FT of the Unit Impulse Function *δ*(*t*):



*δ* (*t*) ⇔ 1

2) FT of the Gate Function: 



The function sin(*x*)/*x* is called sinc(*x*)

rect(*t*/*τ*) ⇔ *τ ⋅* sinc(*ωτ*/2)

3) IFT of  

*ejωot* / 2π ⇔ *δ*(*ω*–*ω*0)

4) FT of 



cos(*ω*0*t*) ⇔ π [*δ*(*ω* – *ω*0) + *δ*(*ω* + *ω*0)]

Similarly,

sin(*ω*0*t*) ⇔ (π/j) [*δ*(*ω* – *ω*0) + *δ*(*ω* + *ω*0)]

**What does Spectrum of a Signal Mean:**

Now we know how to get the FT *G*(*ω*) from a signal *g*(*t*) and how to get *g*(*t*) back from *G*(*ω*) using the IFT formula. But, what does the FT of a signal physically mean? The FT of a signal represents the frequency contents of that signal. That is, what sines or cosines add up together to form the signal. So, it appears that the FT does the same thing as the FS. In fact, that is true. The difference between the FS and the FT is that the FS shows what sines and cosines with frequencies that are multiples of some fundamental frequency *ω*0combine to produce the PERIODIC signal. The periodicity of any signal that the FS simulates (even if the signal we are applying the FS to is not periodic, the FS automatically assumes the periodicity of the signal that is given in the period that we are integrating over) causes the spectrum of the signal to be a discrete–frequency signal that is defined only at multiples of *ω*0. For general signals that are not periodic, the FT (which is a form of the FS) becomes a continuous–frequency signal that is defined for all values of *ω*. So, how much energy (or power) does a sine function, for example, with a specific frequency contribute to a signal that is not periodic? The answer is generally zero. Only if we take a range of frequencies, such as the range of 150 to 160 Hz, we can say that the contribution of the sine waves with frequencies in this range is 10 J (this comes from the fact that the integration of a signal with finite magnitude between the points *t* = 5– (just before 5) to t = 5+ (just after 5) is always zero.

**Properties of the Fourier Transform:**

If *g* (*t*) ⇔ *G* (*ω*) ––––––– (1)

 ––––––– (2)

 ––––––– (3)

**a) Symmetry between the FT and IFT:**

Let s be a time variable and *β* be a frequency variable.



By comparing the term between the brackets with Equation (3) above, we get



*G* (*t*) ⇔ 2*πg*(*–ω*)

**b) Time Scaling:**



If *a* > 0: Let *τ* = *at* 🡺 *dτ* = *a dt* 🡺 *τ = –*∞ → ∞



If *a* < 0: Let *τ* = *at* 🡺 *dτ* = *a dt* 🡺 *τ =* ∞ → –∞





**c) Time and Frequency Reversal:**

, Let *τ* = – *t* 🡺 *dτ* = *– dt* 🡺 *τ =* ∞ → –∞



*g* (–*t*) ⇔ –*G*(–*ω*) (This can also be obtained using (b) above with *a* = –1)

**d) Time Differentiation:**



Since the differentiation is with respect to t and the integration is with respect to *ω*, we can bring derivative inside the integral as



*dg*(*t*)/*dt* ⇔ *jωG*(*ω*) and  *dng*(*t*)/*dtn* ⇔ (*jω*)*nG*(*ω*)

**e) Time Integration:**



Also here, the two integrals are in terms of different variables, so we can switch the order of integration as



Since the period of integration in the inner integral is –∞ ≤ *τ* ≤ *t* , we can insert a unit step function *u*(*t*–*τ*) that is equal to 1 in this region and 0 outside and change the limits of integration to be from –∞ to ∞, which gives

.

By setting *s* = – *τ* , we get



Notice that the inner integral is nothing but the FT of *u*(*t+s*),







**f) Time and Frequency Shifting:**

*g*(*t – t*0) ⇔ *G*(*ω*)⋅*e–jωt*0 and  *g*(*t*) ⋅*ejω*0*t* ⇔ *G*(*ω – ω*0)

**g) Multiplying by a Sinusoid:**

Using the frequency shifting property in (e) above,

*g*(*t*)⋅cos(*ω*0*t*) ⇔ (1/2)[*G*(*ω – ω*0) + *G*(*ω +ω*0)]

*g*(*t*)⋅sin(*ω*0*t*) ⇔ (1/*j*2)[*G*(*ω – ω*0) – *G*(*ω +ω*0)]

**h) Convolution of Two Signals:**

The convolution of two signals *g*(*t*) and *f*(*t*) is defined as 

Similarly,

The FT of the convolution of two signals is the product of the two FTs, and the IFT of the convolution of two signals is the product of the two IFTs

 and 

**References:**

[1] Digital Signal Processing –A Computer based approach (2ND, 3RD and 4TH editions) Sanjit K.Mitra McGraw Hill, 2012.8.

* [2] Digital signal processing written by jhon G. proakis and Dimitris K Manolakis

[2] Website:

[www.ee.ic.ac.uk/naylor/note/DSP4.pdf](http://www.ee.ic.ac.uk/naylor/note/DSP4.pdf).

http://www.docin.com/p-10142638.html

[www.mikroe.com](http://www.mikroe.com)

<https://engineering.purdue.edu>